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Fine structure of alpha decay for odd-oven isotopes of Am, Es, and Md nuclei

G. M. Carmel Vigila Bai[1], R. Nithya Agnes[2]
[1]Rani Anna Government College for women, Tirunelveli, India.
[2]St. John's College, Palayamkottai, India.
Email: rcalvinsamuel@gmail.com

Abstract: Half lives of alpha decay for odd-even isotopes of Am $^{231\text{-}245}$, Es$^{239\text{-}257}$ and Md$^{245\text{-}261}$ nuclei from ground state to ground and excited states of daughter nuclei have been calculated using CYE model. In this work we take into account of both quadruple (β2) and hexadecapole (β4) deformations, as well as the spin- parity of parent and daughter nuclei. The calculated half-lives are compared with available data and are found to be good agreement with each other. The Hindrance factor and branching ratios to the excited states of daughter nucleus also determined. The influence of alpha decay energy and the angular momentum of alpha particle on the half-life time calculation have been studied. The standard deviation value for half-life is computed.

Keywords: Q-value, alpha decay, fine structure

1.Introduction:

The unstable nuclei can decay to stable nuclei via the emission of helium nuclei in alpha decay. The alpha decay has led to most valuable information on nuclear structure and nuclear reactions. The alpha decay theory was formulated by Gamow [1] and independently by Gurney and Condon on the basis of quantum tunnelling. The alpha particles are supposed to be pre-formed within the nucleus and emitted by tunnelling through the potential barrier. The fine structure of alpha decay was formulated by Salomon Rosenblum [2] in 1929. Many theoretical models have been employed to study such alpha fine structure [3-5].In this paper we have studied the fine structure of alpha decay for odd-even nuclei of Am$^{231\text{-}241}$, Es$^{243\text{-}257}$ and Md$^{245\text{-}261}$ isotopes using Cubic plus Yukawa plus Exponential Model [6] by including both quadruple and hexadecapole deformations and the spin- parity effects. We have made our calculations by considering Coulomb, Centrifugal and Yukawa plus exponential potential as interacting barrier for separated fragments and cubic potential for the overlapping region described in section 2. The results and discussion are given in section 3. Finally the conclusions are given in section 4.

2. Cubic plus Yukawa plus Exponential Model(CYEM): In order to study such alpha decay we have developed a model (CYEM) in two sphere approximation in which the zero point vibration energy is explicitly included without violating the conservation of energy and the nuclear inertia mass coefficient dependent on the centre of mass distance has been used.

In this work, the spins and the parities of parent and daughter nuclei are included; the parent nucleus has a quadruple deformation only and the emitted cluster is considered to be spherical. If the daughter nuclei have deformations say quadruple and hexadecapole and if the Q-value of the reaction is taken as the origin, the potential for the post - scission region as the function of the centre of mass distance 'r' of the fragment is given by

$$V(r) = V_c(r) + V_n(r) + \ell(\ell+1)\hbar^2/2\mu r^2 - V_{df}(r) - Q \qquad (1)$$

Here V_c is the coulomb potential between a spheroid daughter and spherical emitted cluster. V_n is the nuclear interaction energy due to finite range effects of Krappe et al[7]; and V_{df} is the change in nuclear interaction energy due to quadruple and hexadecapole deformations in the daughter nuclei, 'r' is the distance between fragment centers , 'ℓ' is the angular momentum, and 'μ' is the reduced mass .The effect of atomic electrons on the energy of alpha particle should also be taken into account.

The energy of alpha particle emitted from nucleus in alpha decay is

$$Q_i = Q_{g.s \to g.s} - E_i^* \qquad (2)$$

Where $Q_{g.s \to g.s}$ is the Q- value for ground state to ground state transition and E_i^* is the excitation energy of daughter nucleus to the i^th state. The ground state to ground state Q-value is given by

$$Q_{g.s \to g.s} = \Delta M_P - (\Delta M_D + \Delta M_\alpha) + [k_1(Z_P^{\beta_1} - Z_D^{\beta_1}) - K_2 Z_C^{\beta_2}]$$

Where Mp, M_D, M_α are the mass excess of parent, daughter and alpha nuclei as tabulated by Audi et al [8].Where the terms in the brackets represent the effect of the screening to the nucleus caused by the surrounding electrons. The quantity $K Z^\beta$ represents the total binding energy of the Z-electrons in the atom, where the values k_1=8.7x10^{-6} M e V and $\beta_1 = 2.517$ for nuclei of $Z \geq 60$ and k_2=13.6 x10^{-6} Me V andβ_2 =2.408 for $Z < 60$ have been found from reported by Huang et al [9].

For a pro late spheroid daughter nucleus with longer axis along the fission direction, Pik - Pichak [10] obtained

$$V_c(r) = \frac{3}{2}\frac{Z_d Z_e e^2 \gamma}{r}\left[\frac{1-\gamma^2}{2}\ln\frac{\gamma+1}{\gamma-1} + \gamma\right] \qquad (3)$$

and for an oblate spheroid daughter nucleus with shorter axis along the fission direction

$$V_c(r) = \frac{3}{2}\frac{Z_d Z_e e^2}{r}\left[\gamma(1+\gamma^2)\arctan\gamma^{-1} - \gamma^2\right] \qquad (4)$$

For the overlapping region, we approximate the potential barrier by a third order polynomial in (r) as suggested by Nix[11]having the form

$$V(r) = -E_v[V(r)_t + E_v]\left\{S_d\left(\frac{r-r_i}{r_t-r_i}\right)^2 - S_e\left(\frac{r-r_i}{r_t-r_i}\right)^3\right\} \qquad (5)$$

$$r_i \leq r \leq r_t$$

Where $r_t = a_e + R_d$

Where z_d, z_e are the atomic numbers of the daughter and emitted cluster respectively, and r_i is the distance between the centers of mass of the daughters and the emitted particle portions in the spheroid parent nucleus.

If the nuclei have spheroid shape, the radius vector R (θ) making an angle θ with the axis of symmetry locating sharp surface of a deformed nuclei is given by ref [12]

$$R(\theta) = R_o\left[1 + \sum_{n=0}^{\propto}\sum_{m=-n}^{n}\beta_{nm}Y_{nm}(\theta)\right] \tag{6}$$

Here R_0 is the radius of equivalent spherical nucleus.

If we consider spheroid deformation β_2, then

$$R(\theta) = R_o\left[1 + \beta_2\left(\frac{5}{4\pi}\right)^{1/2}\left(\frac{3}{2}Cos^2\theta - \frac{1}{2}\right)\right] \tag{7}$$

and if the Nilsson's hexadecapole deformation β_4 is also included in the deformation, then Eq. (6) becomes

$$R(\theta)=R_o\left[1 + \beta_2\left(\frac{5}{4\pi}\right)^{1/2}\left(\frac{3}{2}Cos^2\theta - \frac{1}{2}\right) + \beta_4\left(\frac{9}{4\pi}\right)^{1/2}\frac{1}{8}\left(35cos^4\theta - 30cos^2\theta + 3\right)\right] \tag{8}$$

Expressing the energies in MeV, lengths in fm and time in seconds for calculating the life time of the decay system we use the formula,

$$T = \frac{1.433 \times 10^{-21}}{E_v}\left(1 + \exp(K)\right) \tag{9}$$

The zero-point vibration energy $E_v = \frac{\pi}{2}\frac{\hbar\sqrt{2Q/\mu}}{(C_1+C_2)}$

Where μ is the reduced mass of the system and C_1 and C_2 are the "central" radii of the fragments given by [13]

$$C_i = 1.18A_i^{1/3} - 0.48 \qquad (i = 1,2) \tag{10}$$

The action integral K is given by $K = K_L + K_R$

Where $K_L = \frac{2}{h}\int_{r_a}^{r_t}\left[2B_r\, V(r)\right]^{1/2} dr$ (11)

$K_R = \frac{2}{h}\int_{r_t}^{r_b}\left[2B_r\,(r)V(r)\right]^{1/2} dr$ (12)

The limits of integration r_a and r_b are the two appropriate zeros of the integrand which are found numerically.

3. Results and discussion:

In this work, the half lives of alpha decay for $Am^{231-247}$, $Es^{242-260}$ and $Md^{245-261}$ isotopes have been investigated by using CYEM model. We have made our calculations by including quadrupole and hexadecapole deformations of daughter nuclei which are taken from the tables of Moller et al [14] and spin-parity effects. Table1 gives the Logarithmic half-lives for alpha decay of odd-even nuclei of Am, Es and Md isotopes. Figure 1 gives the plot of logarithmic values of half lives vs Q values. Here we have presented the alpha decay half-lives within the range 10^{-1} s≤ T≤10^{16} s. The angular momentum carried by the alpha particle in a ground state to ground state transition of even-even nucleus is zero. In odd-even or odd-odd nuclei, it could not be equal to zero. In this case, the centrifugal barrier term must be added with the Coulomb barrier. The values of natural angular momentum have been obtained from the usual nuclear spin and parity conservation law,

$$|J_i - J_j| \leq \ell \leq |J_i + J_j| \text{ and } \pi_i/\boldsymbol{\pi}_j = (-1)^{\ell}$$

The computed half-lives of alpha decay are compared with the available data. The calculated half-lives are in good agreement with the available data. Some of the nuclei are found to have long half-lives which indicate the stability of corresponding parent nuclei and lower half –lives value indicates the same corresponding to daughter nuclei. The comparison of calculated partial half-life values with experimental data is shown in figure3.The effect of the centrifugal potential on half-life is studied and tabulated in table3.Table3 gives the logarithmic half –lives of some nuclei for different angular momentum values. It is found that as the angular momentum value increases, the decay rate gets slow down (i.e) the half-life value increases. From fig.3, it is seen that the highest value of branching ratio is found to the transition of $5/2^- \rightarrow 5/2^+$ in ^{241}Am isotope, $7/2^+ \rightarrow 7/2^+$ in^{247}Es isotope and $7/2^- \rightarrow 7/2^+$ in^{257}Md isotopes. Its value going on decreases as we move to the other excited states of daughter nuclei. That means the alpha transitions to the other states are hindered. The standard deviation is estimated using the following expression.

$$\sigma = \sqrt{\frac{1}{(n-1)} \sum_{i=1}^{n} [\log T_i^{\text{theor.}} - \log T_i^{\text{exp.}}]^2}$$

The estimated standard deviation for the half lives by our model is 0.5659 while the same calculated by Ref [15] is 0.56401. The hindrance Factor (HF) is calculated by the formula, HF= $\frac{T_{1/2}^{exp.}}{T_{1/2}^{thero.}}$. The calculated HF is close to unity which shows better result.

Table1.Logarthmic half lives for alpha decay for odd-even isotopes of Am, Es and Md nuclei from ground state to ground state transition including angular momentum and deformation parameter

Parent nuclei	Daughter nuclei	ℓ_{min}	Q(MeV)	Log T(s)				HF_{cal}
				CYEM	Mollar Nix[16]	UMADAC[15]	Expt.[17]	
^{231}Am	^{227}Np	0	7.278	2.50	2.54	1.93	-	-
^{233}Am	^{229}Np	0	7.098	3.18	3.65	3.78	-	-
^{235}Am	^{231}Np	1	6.618	5.35	5.78	7.09	5.17	0.966
^{237}Am	^{233}Np	1	6.238	7.16	7.20	9.01	-	-
^{239}Am	^{235}Np	1	5.965	8.58	7.97	10.41	-	-
^{241}Am	^{237}Np	1	5.680	10.12	10.49	12.02	10.13	1.00
^{243}Am	^{239}Np	1	5.481	11.36	11.88	13.18	11.37	1.00
^{245}Am	^{241}Np	0	5.260	12.57	14.37	12.83	-	-
^{247}Am	^{243}Np	0	4.88	15.16	17.56	15.42	-	-
^{249}Am	^{245}Np	0	4.768	16.07	13.69	-	-	-
^{241}Es	^{237}Bk	3	8.290	1.41	2.11	2.37	-	-

^{243}Es	^{239}Bk	3	8.120	2.00	2.79	3.18	-	-
^{245}Es	^{241}Bk	3	7.960	2.53	4.49	3.73	1.82	0.719
^{247}Es	^{243}Bk	3	7.507	4.26	5.32	5.26	-	-
^{249}Es	^{245}Bk	3	6.984	6.49	7.12	7.58	-	-
^{251}Es	^{247}Bk	0	6.643	6.99	8.93	7.31	5.07	0.725
^{253}Es	^{249}Bk	0	6.784	6.39	6.87	6.65	6.25	0.978
^{255}Es	^{251}Bk	3	6.482	9.01	6.82	10.03	-	-
^{257}Es	^{253}Bk	0	6.090	9.94	11.18	10.24	-	-
^{245}Md	^{241}Es	2	9.032	-0.56	-0.43	-0.92	-	-
^{247}Md	^{243}Es	2	8.812	0.11	1.25	-0.33	0.08	0.727
^{249}Md	^{245}Es	2	8.482	1.19	1.83	0.80	1.34	1.126
^{251}Md	^{247}Es	1	8.011	2.49	3.39	4.23	2.41	0.968
^{253}Md	^{249}Es	1	7.622	3.94	4.81	5.41	-	-
^{255}Md	^{251}Es	2	7.952	3.17	2.70	2.81	3.21	1.013
^{257}Md	^{253}Es	0	7.604	4.14	3.58	5.99	4.30	1.039
^{259}Md	^{255}Es	1	6.891	7.15	7.85	7.91	-	-
^{261}Md	^{257}Es	2	6.802	8.04	10.84	9.99	-	-

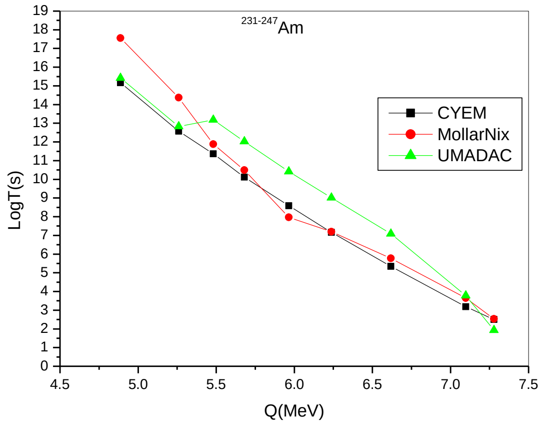
Table2.Logarithmic half lives for alpha decay for odd-even isotopes of Am, Es and Md nuclei from ground state to excited states of daughter nuclei including angular momentum and deformation parameter

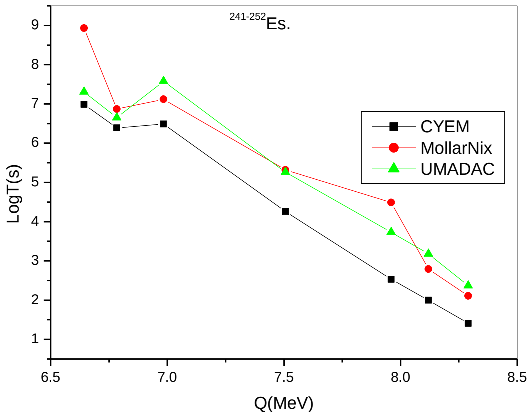
Alpha Transitions	ℓ_{min}	Q(MeV)	LogT(s)			B_{cal}	$B_{expt.}$
			CYEM	CPPM[18]	Expt.[17]		
^{241}Am→^{237}Np							
$5/2^- \rightarrow 5/2^+$	1	5.680	10.12	11.246	12.567	37.8	37
$5/2^- \rightarrow 7/2^+$	1	5.647	10.31	11.442	12.790	24.4	22
$5/2^- \rightarrow 5/2^-$	0	5.621	10.29	11.549	10.207	25.5	84.60
$5/2^- \rightarrow 9/2^+$	3	5.605	11.30	11.955	13.834	2.5	0.02
$5/2^- \rightarrow 7/2^-$	2	5.577	11.04	11.968	11.017	4.5	13.10
$5/2^- \rightarrow 11/2^+$	3	5.550	11.63	12.283	14.137	1.17	0.01
$5/2^- \rightarrow 9/2^-$	2	5.522	11.37	12.307	11.915	2.12	1.66
$5/2^- \rightarrow 11/2^-$	4	5.454	12.70	13.079	13.959	0.09	1.50e-2
$5/2^- \rightarrow 3/2^-$	2	5.413	12.03	12.993	15.436	0.465	4.99e-4
$5/2^- \rightarrow 13/2^-$	4	5.373	13.19	13.581	14.755	0.032	2.39e-3
$5/2^- \rightarrow 7/2^-$	2	5.356	12.38	13.358	15.021	0.208	1.30e-3
$5/2^- \rightarrow 1/2^+$	3	5.348	12.86	13.560	17.137	0.069	9.94e-6
$5/2^- \rightarrow 5/2^-$	0	5.321	12.12	13.436	15.358	0.372	5.97e-4
$5/2^- \rightarrow 5/2^+$	1	5.312	12.35	13.544	15.182	0.22	8.96e-4
$5/2^- \rightarrow 3/2^+$	1	5.310	12.36	13.559	15.658	0.217	2.99e-4
$5/2^- \rightarrow 15/2^-$	6	5.285	12.51	13.684	15.290	0.154	6.99e-4
$5/2^- \rightarrow 11/2^-$	4	5.246	14.89	14.425	15.533	0.217	3.99e-4
$5/2- \rightarrow 9/2^-$	3	5.228	13.63	14.352	15.534	0.012	3.99e-4
$5/2- \rightarrow 7/2^+$	1	5.221	12.93	14.149	15.533	0.059	3.99e-4
^{251}Es→^{247}Bk							
$3/2^- \rightarrow 3/2^-$	0	6.643	6.99	7.954	7.462	82.084	81.02
$3/2^- \rightarrow 5/2^-$	2	6.120	10.17	8.255	8.403	5.423e^{-2}	9.29

$3/2^- \to 7/2^+$	3	6.601	8.24	8.458	8.844	4.616	3.36
$3/2^- \to 7/2^-$	2	6.571	7.89	8.458	7.899	10.33	2.97
$3/2^- \to 9/2^+$	3	6.559	8.44	8.667	8.844	2.913	3.36
$^{257}Md \to ^{253}Es$							
$7/2^- \to 7/2^+$	0	7.604	3.92	4.561	7.566	51.239	0.37
$7/2^- \to 9/2^+$	1	7.559	4.32	4.751	7.696	2.039	0.27
$7/2^- \to 11/2^+$	2	7.524	4.87	5.143	8.121	5.749	0.10
$7/2^- \to 3/2^-$	2	7.498	4.97	5.100	7.997	4.567	0.14
$7/2^- \to 5/2^-$	1	7.465	4.70	5.238	7.719	8.504	0.26
$7/2^- \to 7/2^-$	0	7.423	4.65	5.265	7.821	9.541	95.32

Table3. Logarithmic half lives for alpha decay of some Am, Es and Md nuclei for different ℓ values.

ℓ	Log T(s)		
	Am^{241}	Es^{251}	Md^{257}
0	9.95	12.34	3.92
1	10.12	12.53	4.14
2	10.44	12.89	4.55
3	10.87	13.37	5.10
4	11.37	13.93	5.75
5	11.92	14.55	6041
6	12.51	15.22	7.22
7	13.11	15.91	8.00
8	13.74	16.62	8.81
9	14.37	17.34	9.63





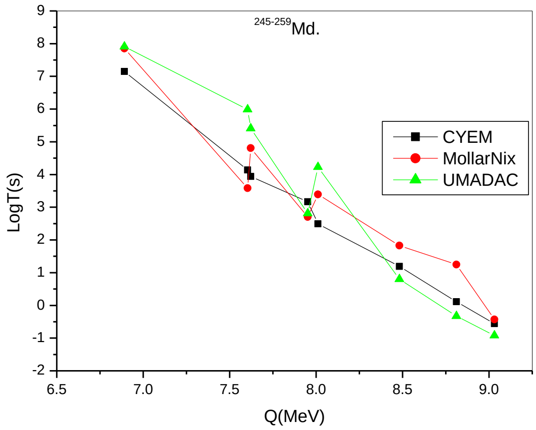


Fig.1. .Logarithmic half lives of alpha decay Vs Q values including deformation effects.

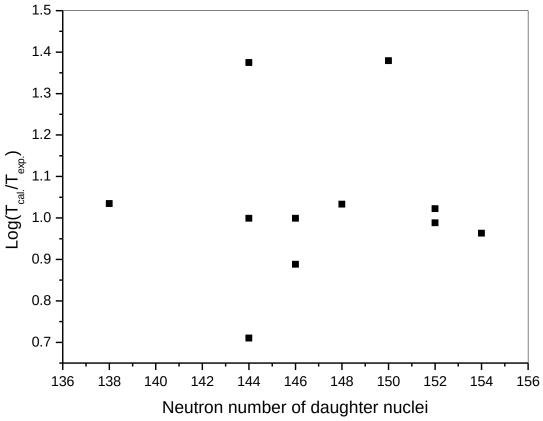


Fig.2.The deviation of calculated $T_{1/2}$ values with the corresponding experimental data.

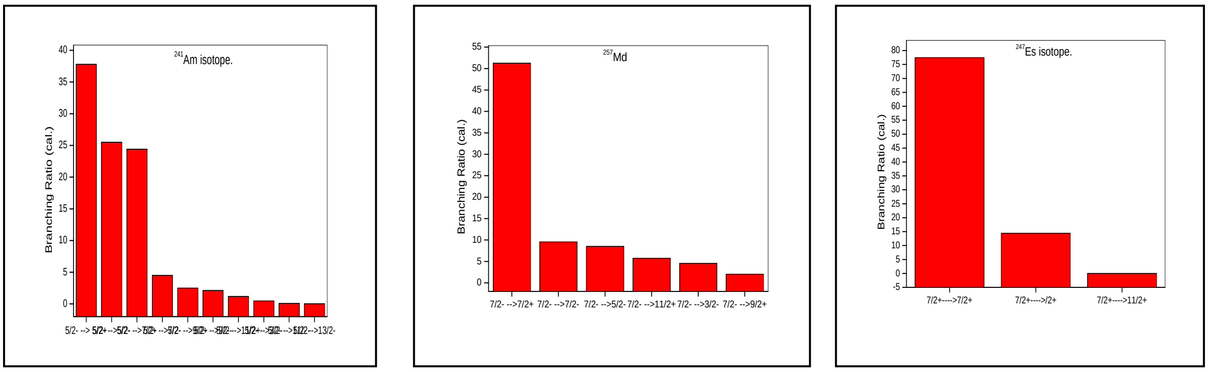


Fig.3.Histogram showing branching ratio for ^{241}Am, ^{247}Es and ^{257}Md isotopes.

4. Conclusions:

We have calculated alpha decay half lives of odd-even nuclei from ground state to ground state and excited states of daughter nuclei using CYE model. The calculated half-lives are compared with the available data. Our results are good agreement with each other. If the Hindrance Factor is between 1 and 4, the transition is called a 'favoured transition'. The HF calculated by our model is found to be unity which shows better result. Due to the Centrifugal potential effect, the half-life value is found to be increased, because it increases the thickness and height of the barrier. The Branching ratio value for some excited states is found to be small. That means the alpha transmissions to those excited states are strongly hindered.

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